Literature review report on state-of-the-art impedance metrology

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This report contains a literature review on state-of-the-art electrical impedance metrology, with specific attention to the frequency range from 100 mHz to 2 MHz. The aim of this report is to provide common background information to the participants to the European Partnership on Metrology project 25IEM05 AIM-EF, *Advancing impedance metrology in an extended frequency range*, in order to ensure that all participants are aware of the latest developments, to identify the specific weak areas where technology is lacking, and to help optimize the technical details for the methods and instrumentation developed in the project. In addition, there are reported also a number of general works on impedance metrology which can be useful to those first approaching the field. A literature review was also produced during the project 17RPT04 VersICaL (2018–2021), *A versatile electrical impedance calibration laboratory based on digital impedance bridges*, funded by the European Metrology Programme for Innovation and Research (EMPIR), and can be found online [1].

To help the reader navigate this report and the article titles, we briefly recall below terminology and abbreviations commonly employed in impedance metrology and in the related fields of circuit theory and radiofrequency. More information can be found in the general works of section 1. Key terms are emphasized.

In the context of linear electrical networks in the sinusoidal steady-state regime, *impedance* is a complex quantity associated to a network element described by voltage and current variables, represented by *complex phasors*. The impedance is then defined as the ratio of a voltage phasor to a current phasor, with possible constraints assigned to the other variables of the element. The real part of the impedance is the *resistance* and the imaginary part is the *reactance*. It is common to distinguish between *DC resistance*, when the resistance is measured at zero frequency, and *AC resistance*, when the resistance is measured at non-zero frequency. The reciprocal of the impedance is the *admittance*. The real part of the admittance is the *conductance* and the imaginary part is the *susceptance*. Depending on the type of network element and the assigned constraints, different impedance definitions are possible: *two-terminal* (2T), *three-terminal* (3T), *four-terminal* (4T), *five-terminal* (5T), *one-terminal-pair* (1TP) or *one-port* (1P), *two-terminal-pair* (2TP) or *two-port* (2P), *four-terminal-pair* (4TP) or *four-port* (4P) and *two-terminal-pair shunt-through*, *two-port shunt-through* or *four-terminal-coaxial*. A *terminal-pair* or *port* (the two terms are used interchangeably) is a pair of element terminals in which the current flowing into one terminal equals the current flowing out of the other terminal. In circuit theory, 2T or 1P impedances are commonly called *driving-point* impedances; driving point impedances and admittances are collectively called *driving-point immittances*; the 4T impedance is also called *transfer impedance* or *transimpedance*; the 4T admittance is also called *transfer admittance* or *transadmittance*; transfer impedances and admittances are collectively called *transfer immittances*. The term *two-port shunt-through* is typically employed in the fields of radiofrequency and power integrity. *Coaxial* terminal-pairs are implemented by a pair of coaxial cylindrical conductors, an inner center conductor and an outer shield. Coaxial impedances are shielded and terminated with coaxial terminal pairs. Coaxiality is typically ensured by external magnetic elements called *current equalizers*, *coaxial chokes* or *common-mode chokes*.

Impedance meters and bridges measure or compare impedances defined at the instrument ports. If a standard is connected to these ports by means of cables, there is a deviation between the impedance defined at the instrument ports and the one defined at the standard ports. This deviation may be corrected by means of *cable corrections* or *cable compensations*.

The unit of impedance is the ohm (Ω). The unit of capacitance, the farad (F), and the unit of inductance, the

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henry (H), are related to the ohm via the unit of time, the second (s): $1\,\Omega = 1\,\mathrm{H/s} = 1\,\mathrm{s/F}$.

The *quantum Hall effect* (QHE) is at the basis of the realization of the ohm in the *International System of Units* (SI). A *quantized Hall resistance* (QHR) standard (QHRS) is a device that produces a quantized resistance from the QHE. The qualifiers DC and AC may be prefixed to QHR. The term *quantum impedance standard* (QIS) is also used for ACQHRS. Three different types of devices have properties suitable for primary resistance and impedance metrology: devices based on gallium arsenide (GaAs) heterostructures, devices based on monolayer graphene and devices based on magnetically-doped topological insulators (TI). However, to date, only GaAs and graphene devices have been employed as QIS.

Impedance bridges compare an impedance ratio to a reference ratio, by *balancing* (equating) the two ratios. In general, impedance bridges can be classified in several ways on the basis of: i) the implemented impedance definition (typically *two-terminal*, *four-terminal*, *two-terminal-pair* or *four-terminal-pair*); ii) the reference ratio quantity (*voltage ratio* or *current ratio*); iii) the bridge architecture (*digitally assisted* (DA), if the reference ratio is mainly defined by a transformer, or *fully digital* (FD), if the reference ratio is completely defined by a digital system); iv) the generator type (*electronic*, if the generator is based on digital electronics, or *Josephson*, if the generator is a programmable Josephson voltage standard (PJVS) or a Josephson arbitrary waveform synthesizer (JAWS)); v) the means used to establish the ratio reading (*generating*, *sourcing* or *DAC-based*, if the reference ratio is determined by the settings of a digital signal generator, or *digitizing*, *sampling* or *ADC-based*, if the reference ratio is determined from digitized samples).

In impedance bridges, the constraints assigned by impedance definitions are usually imposed by auxiliary networks (e.g. *Kelvin arm*, *Wagner ground/earth*), which require additional balances. Certain constraints can be eliminated by linear voltage combinations performed by *combining networks*.

LCR meters, *impedance meters* and *impedance analyzers* directly measure 4T or 4TP impedances and are typically based on *autobalancing bridges* or on the *IV* method.

[1] M. Ortolano and L. Callegaro. *VersICaL bibliography on digital impedance bridges*. 2018. [URL]. Online.

1 General works on impedance metrology

This section collects general works, like books and review articles, about impedance metrology. It also contains a number of related works on impedance definition, current equalizers, bridge balancing algorithms and impedance measurement comparisons.

1.1 Books and handbooks

Books [2] and [3] are classic works on impedance bridges; [4] extends and updates [3]; [4] and [5] present, with complementary approaches, up-to-date material on impedance measurement methods and impedance standards. Both [4] and [5] have extensive bibliographies.

[2] B. Hague. *Alternating current bridge methods*. 6th ed. London, UK: Pitman Publishing Ltd, 1971. ISBN: 0-273-40291-9.
[3] B. P. Kibble and G. H. Rayner. *Coaxial AC bridges*. Bristol, UK: Adam Hilger Ltd, 1984. ISBN: 0-85274-389-0.
[4] S. Awan, B. Kibble and J. Schurr. *Coaxial electrical circuits for interference-free measurements*. Electrical Measurement Series. 2011. ISBN: 978-1-84919-069-5. [DOI].
[5] L. Callegaro. *Electrical Impedance: Principles, Measurement, and Applications*. CRC Press, 2013. ISBN: 978-1-4398-4910-1. [DOI].

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1.2 Review articles

A classic reference for the history of impedance measurements is [6], written by a former scientist at General Radio. Reference [7] is a good starting point for a review on modern impedance bridges, including digital bridges, with a comprehensive bibliography. Digital bridges are also introduced in [5]. Reference [8] is a brief review about traceable impedance measurements for non-specialists. A rather technical analysis of error sources in fully-digital bridges, with a review of the operating principles, can be found in [9]. Reference [10] reviews the metrology of impedance at high frequency, discussing also the connection between scattering matrix, open-circuit impedance matrix, short-circuit admittance matrix and the 4TP impedance, calculable standards and cable corrections. The principles of operation and the usage of commercial LCR meters and impedance analyzers are described in [11]. A number of techniques for the estimation of the local impedance at a point of common coupling in a utility power network are reviewed in [12]. The usage of GaAs devices as QIS is reviewed in [13–17]. The usage of graphene devices as QIS is reviewed in [18].

[6] H. P. Hall. *A history of impedance measurements*. 1992. [URL].

[7] F. Overney and B. Jeanneret. 'Impedance bridges: from Wheatstone to Josephson'. In: *Metrologia* 55 (2018), S119–S134. [DOI].

[8] L. Callegaro. 'Traceable measurements of electrical impedance'. In: *IEEE Instrum. Meas. Mag.* 18 (2015), pp. 42–46. [DOI].

[9] M. Ortolano et al. 'A comprehensive analysis of error sources in electronic fully digital impedance bridges'. In: *IEEE Trans. Instrum. Meas.* 70 (2021), p. 1500914. [DOI].

[10] L. Callegaro. 'The metrology of electrical impedance at high frequency: A review'. In: *Meas. Sci. Technol.* 20.2 (2009), p. 022002. [DOI].

[11] Keysight Technologies. *Impedance Measurement Handbook*. 2026.

[12] M. K. De Meerendre et al. 'Review of local network impedance estimation techniques'. In: *IEEE Access* 8 (2020), pp. 213647–213661. [DOI].

[13] F. Overney et al. 'The quantized Hall resistance: towards a primary standard of impedance'. In: *Metrologia* 43.5 (2006), pp. 409–413. [DOI].

[14] J. Schurr et al. 'The ac quantum Hall effect as a primary standard of impedance'. In: *Metrologia* 44.1 (2007), pp. 15–23. [DOI].

[15] F. J. Ahlers et al. 'Compendium for precise ac measurements of the quantum Hall resistance'. In: *Metrologia* 46.5 (2009), R1–R11. [DOI].

[16] J. Schurr et al. 'The quantum Hall impedance standard'. In: *Metrologia* 48.1 (2011), pp. 47–57. [DOI].

[17] J. Schurr, F. Ahlers and B. P. Kibble. 'The ac quantum Hall resistance as an electrical impedance standard and its role in the SI'. In: *Meas. Sci. Technol.* 23 (2012), p. 124009. [DOI].

[18] L. Callegaro et al. *Good practice guide on the graphene-based AC-QHE realization of the farad*. 2022. ISBN: 978-88-945324-3-2. [URL].

1.3 Impedance definitions, combining networks and cable corrections

Impedance definitions, combining networks and cable corrections are generally discussed in [4] and [5]. The 4TP impedance definition was first introduced in [19]. Reference [20] presents a review of the concept and purpose of the Wagner ground. The effects of high-resistance cables, like those used to connect QHR standards in cryogenic environments, is discussed in [21, 22]. Cable corrections for high-frequency measurements are discussed in [23]. Reference [24] derives a relation between 4TP impedance and scattering parameters, enabling the frequency dependence of impedance standards to be determined from VNA measurements without requiring an equivalent lumped-element model.

[19] R. D. Cutkosky. 'Four-terminal-pair networks as precision admittance and impedance standards'. In: *IEEE Trans. Commun. Electr.* 83 (1964), pp. 19–22. [DOI].

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[20] H. H. Wolff. 'Wagner-earth and other null instrument capacity neutralizing circuits'. In: *Rev. Sci. Instrum.* 30.12 (1959), pp. 1116–1122. [DOI].
[21] J. Melcher. 'Systematic errors of long, thin coaxial cables for the connection of four terminal-pair devices'. In: *IEEE Trans. Instrum. Meas.* 45.1 (1996), pp. 339–341. [DOI].
[22] B. Chenaud et al. 'Influence of coaxial cable resistance on impedance measurements at low frequency and guidelines for phase determination of ac electrical signals'. In: *Rev. Sci. Instrum.* 94.11 (2023), p. 115112. ISSN: 0034-6748, 1089-7623. [DOI].
[23] M. Agustoni et al. 'Cable correction model for HF resistance standards comparisons'. In: *IEEE Trans. Instrum. Meas.* 74 (2025), p. 1503507. [DOI].
[24] L. Callegaro and F. Durbiano. 'Four-terminal-pair impedances and scattering parameters'. In: *Meas. Sci. Technol.* 14.4 (2003), p. 523. [DOI].

1.4 Current equalizers

The first systematic discussion on the usage of current equalizers in impedance bridges can be found in [25]. This reference also presents the concept of active current equalizer. General presentations of current equalizers can be found in [4], which also discusses placement and effectiveness verification, and in [5]. The effects of unequalized currents in impedance bridges are discussed in [26–28].

[25] D. N. Homan. 'Applications of coaxial chokes to a-c bridge circuits'. In: *J. Res. Natl. Bur. Stand. Sec. C* 72C.2 (1968), p. 161. [DOI].
[26] J. Schurr and J. Melcher. 'Unequalized Currents in Coaxial AC Bridges'. In: *IEEE Trans. Instrum. Meas.* 53 (2004), pp. 807–811. [DOI].
[27] G. A. Kyriazis, R. T. B. Vasconcellos and J. Melcher. 'Unequalized currents in two-terminal-pair coaxial capacitance bridges'. In: *Proc. XVIII IMEKO World Congress*. XVIII IMEKO World Congress. Rio de Janeiro, Brazil, 2006. [URL].
[28] Y. A. Sanmamed et al. 'Evaluation of the influence of the unequalized currents in the inductance bridge at CEM'. In: *Proc. XX IMEKO World Congress*. Busan, Republic of Korea, 2012. [URL].

1.5 Bridge balancing algorithms

Impedance bridges operate by *balancing*: the voltage or the current at specific terminal-pairs of the bridge network are nulled by adjusting a corresponding number of balancing elements (voltages, currents or impedances) of the network. The main balance provides the bridge reading; auxiliary balances achieve the impedance definition conditions, or the correct operation of the bridge elements. Strategies to efficiently achieve the balance condition are the subject of the papers of this section.

[29] M. Dutta et al. 'An application of an LMS adaptive algorithm for a digital AC bridge'. In: *IEEE Trans. Instrum. Meas.* IM-36 (1987), pp. 894–897. [DOI].
[30] S. S. Awad, N. Narasimhamurthi and W. H. Ward. 'Analysis, design, and implementation of an AC bridge for impedance measurements'. In: *IEEE Trans. Instrum. Meas.* 43 (1994), pp. 894–899. [DOI].
[31] P. Holmberg. 'Automatic balancing of linear AC bridge circuits for capacitive sensor elements'. In: *1994 IEEE Instrum. Meas. Technol. Conf.* Hamamatsu, Japan, 1994. [DOI].
[32] P. Holmberg. 'Automatic balancing of linear AC bridge circuits for capacitive sensor elements'. In: *IEEE Trans. Instrum. Meas.* 44 (1995), pp. 803–805. [DOI].
[33] J. Q. Zhang, S. J. Ovaska and X. Zhao. 'A novel fast balance technique for the digital AC bridge'. In: *IEEE Trans. Instrum. Meas.* 47 (1998), pp. 371–377. [DOI].
[34] A. Carullo et al. 'Automatic compensation system for impedance measurement'. In: *IEEE Trans. Instrum. Meas.* 52 (2003), pp. 1239–1242. [DOI].
[35] L. Callegaro. 'On strategies for automatic bridge balancing'. In: *IEEE Trans. Instrum. Meas.* 54 (2005), pp. 529–532. [DOI].

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[36] N. K. Das, T. Jayakumar and B. Raj. 'Noniterative digital AC bridge balance'. In: *IEEE Trans. Instrum. Meas.* 59 (2010), pp. 3058–3060. [DOI].
[37] Y. Khoma et al. 'Synthesis of dynamic errors correcting algorithm for auto-balancing bridge methods'. In: *IEEE Access* 8 (2020), pp. 165307–165318. [DOI].
[38] M. Huang et al. 'A fast digital balance algorithm for auto-balancing bridge'. In: *2025 IEEE 17th International Conference on Electronic Measurement & Instruments (ICEMI)*. 2025, pp. 223–228. [DOI].

1.6 Comparisons

A major goal of metrology is to establish equivalence of measurements through comparisons. Official key and supplementary impedance comparisons are organised at European level by EURAMET, and at international level by the Consultative Committee for Electricity and Measurements (CCEM) of the Metre Convention, and reported in the BIPM website. New developments in impedance metrology have been tested also by comparison carried out as research projects. A first comparison between Josephson impedance bridges was reported in [39]. References [40–42] report comparisons between a new generation of digital and Josephson bridges.

[39] G. Eklund et al. 'A comparison of the Josephson impedance bridges of PTB and SP'. In: *2016 Conf. Precis. Electromagn. Meas. (CPEM 2016)*. Ottawa, Canada, 2016, pp. 1–2. [DOI].
[40] M. Ortolano et al. 'An international comparison of phase angle standards between the novel impedance bridges of CMI, INRIM and METAS'. In: *Metrologia* 55 (2018), pp. 499–512. [DOI].
[41] M. Marzano et al. 'PTB–INRIM comparison of novel digital impedance bridges with graphene impedance quantum standards'. In: *Metrologia* 59 (2022), p. 065001. [DOI].
[42] M. Ortolano et al. 'An International trilateral comparison among the newest generations of digital and Josephson impedance bridges'. In: *IEEE Trans. Instrum. Meas.* 74 (2025), p. 1501009. [DOI].

1.7 Component nonlinearity and harmonic distortion

Voltage-dependent resistance can introduce harmonic distortion and amplitude-dependent errors in precision impedance-measurement systems, making resistor nonlinearity an important consideration in component selection and characterisation. A modified Wheatstone bridge driven by a single ground-referenced source determines resistor voltage-nonlinearity coefficients from second- and third-harmonic measurements, with experimental validation and an uncertainty analysis [43].

[43] V. Janásek et al. 'A modified Wheatstone bridge for high-precision characterisation of resistor nonlinearity'. In: *IEEE Trans. Instrum. Meas.* (2026), p. 1502709. [DOI].

2 Fully digital and digitally assisted impedance bridges for extended frequency range

The development of FD and DA impedance bridges has significantly expanded the flexibility of precision impedance measurements and enabled new approaches to impedance traceability. In conventional impedance metrology, highly accurate voltage ratios are commonly realized using inductive voltage dividers (IVDs) or transformer-based ratio devices. These techniques are particularly well established at low and audio frequencies. With increasing frequency, however, the frequency dependence of the ratio devices and the growing influence of stray impedances make it progressively more difficult to maintain high accuracy. Therefore, digital signal generation, synchronous sampling, and numerical determination of complex voltage ratios provide an attractive alternative for extending the usable frequency range of impedance bridges.

In terms of frequency range, the relevant literature can be divided into two main groups. The first group comprises low- and audio-frequency FD or DA bridges, generally operating from tens of hertz up to several or tens of kilohertz. The second group comprises high-frequency (HF) bridges, considered here primarily for frequencies above 20 kHz and extending into the hundreds of kilohertz and megahertz ranges. The interval between approximately 10 kHz and 20 kHz may be regarded as a transition region. This distinction is important because

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the dominant limitations of bridge measurements change substantially when the frequency is increased beyond the audio-frequency range.

2.1 Low- and audio-frequency digital bridges

Most of the development of digital impedance bridges has taken place in the low- and audio-frequency range. Early work already demonstrated the use of digital AC bridges for impedance-to-frequency conversion [44]. Furthermore, digital generation of bridge signals was demonstrated using a dual-channel RAM/DAC sinewave generator with adjustable phase and amplitude for high-precision admittance comparisons [45]. Subsequent developments introduced digital generation and synchronous acquisition of the bridge signals. Several research groups demonstrated digital impedance bridges with automatic source-ratio balancing, allowing the complex source ratio to be adjusted numerically rather than by manual balancing [46–50].

A further important development was the introduction of digitally assisted and fully digital multi-terminal bridges. Digital implementations of 2TP, 4TP, and 5T impedance bridges were subsequently demonstrated [47, 51–55]. In particular, the reconfigurable 4TP architecture demonstrated that a common platform could support both digitally assisted and fully digital operation [54, 55]. Reference [56] describes and validates a fully digital 4TP bridge combining digitally synthesized excitation with synchronous PXI sampling to compare impedance across the complex plane from 20 Hz to 20 kHz, identifying digitizer nonlinearity as a limitation for unequal-ratio comparisons.

Other developments addressed direct comparisons between different impedance types. A digital compensation bridge for R-C comparisons was developed in [57], while a versatile electrical impedance calibration laboratory based on a digital impedance bridge was demonstrated in [58]. These approaches are important for modern impedance metrology because they reduce the need for long chains of intermediate standards and provide a direct route towards flexible impedance comparisons. A digital four-arm bridge incorporating source, detector and main balances, achieving a combined standard uncertainty of 5×10^{-9} for comparison of a 100 kΩ resistor with a 1 nF capacitor near 1592 Hz is demonstrated in [59].

The maturity of the technology in the considered frequency range is also demonstrated by its application to primary impedance metrology. Direct comparisons between capacitance standards and QHRS have been investigated using digital bridge techniques [60]. Related work and international comparisons have demonstrated the use of fully digital bridges for the realization and dissemination of impedance standards, including capacitance and inductance, as well as for international comparisons of phase-angle standards [40–42, 61, 62]. Consequently, low- and audio-frequency digital bridges can now be regarded as a relatively mature technology, with demonstrated precision suitable for primary and national metrology applications.

Nevertheless, extension towards the upper end of the audio-frequency range already reveals limitations that become increasingly important at higher frequencies. A broadband, fully automated digitally assisted coaxial bridge operating from 60 Hz to 50 kHz was developed in [63]. For 1 : 1 comparisons of calculable AC resistors, agreement between the calculated and measured frequency dependence was better than 5×10^{-8} at frequencies up to 5 kHz and better than 10^{-7} up to 20 kHz, whereas the deviation increased to 10^{-6} at 50 kHz. This behavior clearly demonstrates that moving from audio frequencies into the HF range is accompanied by a steep increase in non-ideal contributions, such as electromagnetic coupling, parasitic capacitances, and skin effect losses, which can no longer be neglected.

2.2 High-frequency impedance bridges

The state of the art in precision impedance metrology becomes considerably less mature when fully digital bridge architectures are extended beyond the audio-frequency range. It should be noted, however, that traceable impedance measurements at frequencies well above 20 kHz had already been demonstrated using conventional coaxial and 4TP bridges. The main challenge in this frequency range is therefore not the extension

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of impedance measurements itself, but the achievement of low measurement uncertainties while maintaining a digitally assisted or fully digital architecture.

An important development in high-frequency impedance metrology was the development of a 4TP coaxial AC bridge operating from 100 kHz to 1 MHz [64]. The bridge employed an inductive voltage divider as the reference ratio element and enabled 1 : 1 and 10 : 1 resistance and capacitance ratio measurements with uncertainties of a few parts in 10^5. This work demonstrated the feasibility of traceable impedance comparisons in the MHz range and, at the same time, highlighted the increasing importance of frequency-dependent effects in the bridge components and interconnections. Such conventional HF bridges provide an important reference point for the subsequent development of digital impedance bridges.

A different approach to high-frequency impedance-ratio measurements was presented in [65], where a 2TP impedance bridge was developed for capacitance-ratio measurements at frequencies up to 1 MHz. Instead of using an inductive voltage divider as the primary ratio element, the bridge employed two voltage sources and a phase-control unit to generate the required complex source ratio electronically. This approach provided an alternative to the transformer-based ratio element and offered a practical means of generating the required voltage ratio for high-frequency capacitance measurements. Although the system was not a fully digital impedance bridge in the sense of later digital sampling architectures, the use of electronically controlled voltage sources represented an important step towards replacing frequency-dependent analogue ratio standards by electronically generated signal ratios

The subsequent development of DA and FD impedance bridges introduced digital signal generation and sampling as fundamental elements of the measurement process. In this context, a reconfigurable 4TP impedance ratio bridge capable of operating in both DA and FD configurations was presented in [55]. In the DA mode, an inductive voltage divider could be used as a reference ratio element, whereas the FD configuration relied on digitally generated voltage signals. This architecture demonstrated that functions traditionally performed by analogue ratio standards could, in principle, be implemented using electronically controlled sources and digital signal processing. However, its operating-frequency range did not itself constitute an extension of fully digital impedance metrology into the MHz region.

A significant extension of digital impedance metrology into the HF range was subsequently demonstrated by a 4TP digital sampling impedance bridge operating at frequencies up to 1 MHz [66]. The system was optimized for measurements of low impedances and current shunts, covering impedance values from 50 mΩ to 100 kΩ at currents of up to 3 A. The bridge incorporated a dual-channel multiplexer with multi-stage guarding and a fully automated nonlinearity correction procedure based on a pair of calculable resistors. It achieved an expanded uncertainty of the impedance magnitude of approximately 10 μΩ/Ω at 100 kHz and 50 μΩ/Ω at 1 MHz for impedance ratios up to 1 : 16, together with a phase-shift uncertainty of approximately 360 μrad/MHz. This work demonstrated that digital sampling techniques can support high-accuracy impedance comparisons well into the MHz range, but also illustrated the increasing difficulty of maintaining very low uncertainties as frequency increases.

The limitations of digital bridge architectures at HF were examined more explicitly in [67], where a digital coaxial impedance bridge was evaluated up to 100 kHz in both DA and FD configurations. At 100 kHz, the FD configuration could not achieve an accuracy better than 10^{-4} even for a 1 : 1 impedance comparison, whereas the DA configuration provided better performance. Voltage injection was subsequently introduced to improve the FD operation, and 10 : 1 capacitance-ratio measurements were demonstrated with an accuracy of approximately 10^{-5} at 100 kHz. These results illustrate an important limitation of extending fully digital bridge architectures towards HF: replacing an analogue ratio standard by digitally generated signals does not, by itself, eliminate the frequency-dependent error mechanisms that become increasingly significant at higher frequencies.

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2.3 Summary

The literature demonstrates a clear difference in technological maturity between the two frequency regions. In the low- and audio-frequency range, digital and digitally assisted bridges have reached high maturity and are routinely applied to primary impedance realizations, international comparisons, and traceability chains [40–42, 44, 45, 47, 50–55, 57, 58, 61, 62, 68]. In contrast, HF implementations remain far more specialized, with high precision typically reported only up to 100 kHz or 1 MHz [64, 66, 67, 69].

Nevertheless, performance limitations already begin to emerge toward the upper end of the audio-frequency range, as illustrated by automated digitally assisted coaxial bridges operating up to 50 kHz [63]. This behavior is driven by several compounding non-ideal effects that become critical above 20 kHz:

- Frequency-dependent source and signal path errors: The amplitude and phase stability of voltage generators and analogue front-ends degrade with frequency, introducing ratio errors.
- Hardware parasitic impedances: Stray capacitances, line inductances, and residual screen/skin effects in the bridge network create additional phase shifts and loading errors.
- Standard non-idealities: The impedances of the standards themselves become strongly frequency-dependent, requiring complex lumped/distributed electrical models rather than ideal nominal values.
- Analogue front-end bottlenecks: System performance is often limited not by digital synthesis algorithms, but by finite detector input impedance, source output impedance, and common-mode coupling.
- Topology and ratio sensitivity: Measurement uncertainty increases rapidly when operating at extreme impedance ratios or with uncompensated measurement topologies.

These limitations justify continued research into extending the frequency range of impedance bridges. The key challenge is not simply boosting generator or ADC sampling rates, but designing measurement architectures and error-compensation methods that preserve traceability when systemic parasitic properties dominate. Promising avenues include digitally controlled multi-source topologies, dynamic error-injection techniques, fully characterized 4TP coaxial structures, and integrated electro-numerical modeling.

Ultimately, the available literature—from the limits observed in upper audio-frequency coaxial bridges [63] to the performance degradation reported near 100 kHz [67]—highlights a persistent gap between audio-frequency precision and HF capability. This underscores the need for further work toward a broadly applicable, traceable, and low-uncertainty digital impedance bridge covering the transition from audio frequencies into the high-frequency range.

[44] H. Schollmeyer. ‘A digital AC bridge as an impedance to frequency converter’. In: *IEEE Trans. Instrum. Meas.* 34 (1985), pp. 389–392. [DOI].
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[46] L. Callegaro, V. D’Elia and B. Trinchera. ‘Realization of the farad from the dc quantum Hall effect with digitally assisted impedance bridges’. In: *Metrologia* 47 (2010), pp. 464–472.
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3 Impedance measurement methods in very low and power-line frequency range

This section reviews the state of the art methods and techniques for electrical impedance measurements in the very low frequency (VLF) range, spanning from millihertz up to a few hertz, and in the power-line frequency range, around 50 Hz/60 Hz and its immediate spectral neighborhood. Impedance metrology in these frequency regimes is increasingly critical for key applications, including the characterization of energy storage systems (such as battery electrochemical impedance spectroscopy and supercapacitors), power grid impedance estimation, transmission line parameter identification, and low noise sensor instrumentation.

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3.1 Impedance measurement methods in very low frequency range

The characterization of impedance at low frequency is paramount for electrochemical devices and bio-applications. To address the high cost and complexity of dedicated commercial VLF instrumentation, Scandurra et al. [70] presented a portable, highly versatile impedance meter operating in the sub-hertz frequency region. Their system utilizes a commercial audio sound card interfaced with a dedicated DC-coupled analog front-end for signal generation and digital acquisition, demonstrating accurate characterization of polypyrrole supercapacitors and bioimpedance samples. When performing VLF Electrochemical Impedance Spectroscopy (EIS) on batteries (such as Li-ion or NiMH cells), measurement reliability is severely degraded by long test durations. Scott and Hasan [71] investigated battery impedance measurements in the ultra-low frequency regime (100 µHz to 1 mHz), highlighting how State of Charge (SoC) drift and voltage instabilities during excitation lead to significant errors in commercial analyzers. To overcome these issues, they proposed a current excitation scheme combined with a reduced four-parameter fractional-capacitor equivalent circuit model. Focusing on diffusion kinetics, Oldenburger et al. [72] investigated the low frequency Warburg impedance of Li-ion cells down to 0.1 mHz in the frequency domain. Their findings demonstrated that the Warburg response at low frequencies is highly sensitive not only to the SoC, but also to signal amplitude, current direction, rest time, and open-circuit voltage (OCV) hysteresis. They recommended time-domain pulsed measurement techniques or specific correction protocols to avoid systematic errors caused by prolonged excitation. To provide traceable confidence intervals for battery characterization, Moradpour et al. [73] established a comprehensive uncertainty evaluation framework for battery EIS in the 100 mHz to 5 kHz range. Identifying contact/fixture repeatability and measurement noise as dominant error sources, they propagated measurement uncertainties onto the complex Nyquist plane using METAS UncLib, yielding confidence ellipses for each measured frequency point. Finally, precision VLF and low frequency noise measurements require highly stable, ultra-low noise biasing sources. Routoure et al. [74] designed and characterized a quasi-ideal, low noise DC current source specifically dedicated to four-probe low frequency noise and impedance characterization. The proposed source provides high output impedance and minimal background current noise, establishing an essential building block for high accuracy low frequency metrological chains.

3.2 Impedance measurement methods in power-line frequency range

Traditionally, the grid impedance was mainly determined by passive components such as cables, transformers and conventional loads; nevertheless, the increasing penetration of inverter-based devices (photovoltaic panels, battery storage systems and EV chargers) and power-electronic equipment have transformed low-voltage (LV) grids into active networks. Consequently, the grid impedance is now strongly frequency-dependent, operating-point-dependent and time-varying, including variations within the fundamental-frequency cycle [75]. Therefore, conventional impedance approximations derived from short-circuit impedance become increasingly inaccurate as frequency increases [75].

These effects are also relevant in the supraharmonic range above 2 kHz, where impedance can be dominated by the input characteristics and control behaviour of equipment connected in the vicinity of the measurement point. Several measurement campaigns aimed at characterizing the frequency-dependent grid impedance up to 150 kHz have been conducted [76–80], but further initiatives covering a wider variety of grid topologies and loading conditions are still required to determine representative impedance models.

Although the frequency-dependent impedance of LV distribution grids has become an increasingly relevant parameter for power quality, electromagnetic compatibility, resonance assessment and power-line communication (PLC), there is currently no standardized measurement methodology. Different invasive and non-invasive techniques have been proposed, based on single-frequency sweeps, broadband excitation, impulse injection or naturally occurring disturbances. However, their equivalence, measurement uncertainty and applicability remain insufficiently established, which obstructs the direct comparison of results.

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These issues also question the representativeness of traditional standardized artificial networks for laboratory conditions. Existing definitions of Artificial Mains Networks (AMNs) and Line Impedance Stabilization Networks (LISNs) provide outdated impedance values, and do not reproduce the complex frequency dependence and sub-cycle variations found in modern distribution grids. The current LV grid impedance characteristics should be reconsidered to update these reference laboratory devices. The definition of a more realistic reference impedance boxes is also required to validate measurement instruments and compare assessing methods [76].

Impedance measurement around power-line frequencies (50 Hz/60 Hz) is fundamental for monitoring power quality, ensuring control stability in grid-connected converters, and assessing power system infrastructure. For real-time estimation of grid impedance at the point of common coupling (PCC), Mohammed et al. [81] performed a comparative analysis of two active, inverter-based estimation techniques. They evaluated single frequency non-interfering current injection (at 75 Hz) against wideband Pseudo Random Binary Sequence (PRBS) injection. The study highlighted the trade-offs between estimation speed, fundamental frequency accuracy, and total harmonic distortion (THD) impact on the grid. Regarding power line infrastructure monitoring, Gerez et al. [82] proposed a parameter estimation method to determine the frequency-dependent parameters (resistance, inductance, capacitance, and conductance) of high-voltage transmission lines. Using synchronized voltage and current phasor measurements (such as PMU data) at both line ends and modeling the line as a two-port network, an optimized least-squares formulation allows for accurate dynamic tracking of line parameters across power frequencies and harmonics. Lastly, Passerini and Tonello [83] explored the dual use of Power Line Communication (PLC) devices as distributed impedance sensors across power networks. By extracting the high-frequency and power-line network impedance (Z_{PL}), reflection coefficients (ρ), and transfer functions (T), their work demonstrates how online grid impedance mapping can be leveraged for grid topology identification, fault detection, and cable degradation monitoring.

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4 Josephson and quantized Hall resistance based impedance bridges

This section will cover different impedance bridges based on Josephson voltage standards as high performance quantum based sources for the voltage definition. Moreover, impedance bridges applying the QHR as a quantum-based reference impedance will be listed. The latter can be combined many kinds of impedance bridges (IVD, digital or Jospehson).

4.1 Systems based on programmable Josephson voltage standards (PJVS)

First attempts made on 2TP impedance bridges with quantum based voltage sources were made using programmable Josephson voltage standards up to 10 V. First this systems were used for 1 : 1 resistance and capacitance ratios using rectangular waves and voltages of 1.2 V [84]. Later on also 1 : 10 ratios of up to 10 V [85], approximated sine waves [86] and 4TP definitions [85] were realized. The use of square waves limits the high performance to the measurement of like impedance. For quadrature measurements the achievable uncertainties are limited to parts in 10^6 because the bridge is balanced at the signal frequency but not at higher harmonics which occurs due to the use of square waves [87].

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4.2 Systems based on pulse driven Josephson voltage standards (ACJVS or JAWS)

The limitation concerning quadrature measurements can be solved by the use of pulse-driven Josephson voltage standards. These versatile voltage standards are able to generate arbitrary signals and hence are often called Josephson Arbitrary Waveform Synthesizer (JAWS). The harmonic content of the generated signal is outstanding low such that harmonics are no limitation at all. Up to now two systems have been implemented of which one is setup by METAS [88–91] and the other one by PTB [92–96]. Both systems can be used for arbitrary impedance comparison. These systems were already compared to IVD based and digital impedance bridges [41, 42] or against each other [97]. Moreover, impedance bridge can also be used to investigate Josephson voltage standards as for example in [98].

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4.3 Systems employing AC quantized Hall resistance standards

The study of the AC response of QISs at low frequency up to the kHz range has started for GaAs devices shortly after the adoption of the quantized Hall resistance as a conventional representation of the ohm [99–105]. In subsequent years, efforts have been made to better understand the causes of the AC response of GaAs QISs and to limit the frequency dependence [106–112], and to develop transformer-ratio bridges for the direct comparison of resistance and capacitance standards to GaAs QISs in the kHz frequency range [113, 114]. In more recent years, graphene QISs and digital and Josephson-based bridges simplified the implementation of impedance bridges employing QISs and allowed the extension of the operating frequency range up to tens of kHz [41, 42, 92, 94, 95].

[99] F. Delahaye. ‘An AC-bridge for low-frequency measurements of the quantized Hall resistance’. In: *IEEE Trans. Instrum. Meas.* 40.6 (1991), pp. 883–888. [DOI].
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[106] F. Overney, B. Jeanneret and B. Jeckelmann. ‘Effects of metallic gates on ac measurements of the quantum hall resistance’. In: *IEEE Trans. Instrum. Meas.* 52 (2003), pp. 574–578. ISSN: 0018-9456. [DOI].
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5 Calculable impedance standards for extended frequency range

Impedance metrology is primarily concerned with the practical realization and dissemination of the units of impedance—ohm, farad and henry—and their corresponding scale realizations. Because these units are interrelated, the realization of any one of these units can, in principle, be used to establish the other two by means of appropriate AC bridge techniques [2]. Historically, several calculable artefacts have served as primary standards for this purpose [115–118]. In the early twentieth century, the henry was realized using a calculable mutual inductor [119]. Subsequently, calculable capacitors became the preferred route [120], largely because their capacitance can be derived with high accuracy from a single mechanical dimension. Thus, calculable impedance standards can be divided into two complementary types. The first, absolute calculable standards determine an electrical unit directly from dimensional measurements and electromagnetic theory. Examples include the Thompson–Lampard calculable capacitor [120] and classical calculable self- or mutual-inductance standards. The second type are impedance standards that begin with a known traceable direct current (DC) or low-frequency value and their relative value at a higher frequency are calculable or have a finite frequency-response-coefficient (FRC) based on fundamental geometrical and electromagnetic effects. A prime example of such standards was detailed in 2007 by Awan and Kibble [121] reported a universal geometry, where the same electromagnetic concept can be applied to resistance, capacitance and inductance (LCR) standards, providing a route to calculable impedance standards from DC to at least 100 MHz frequencies. Numerous impedance standards with a variety of designs have been published over the last seven decades, we highlight some key examples here.

Since the 1990s, following the introduction of quantum electrical standards, the practical realization of the volt and the ohm has relied on conventional values of the Josephson and von Klitzing constants [122]. The farad and the henry can subsequently be derived from this conventional realization of the ohm. After the redefinition of the International System of Units (SI) [123, 124], this representation of the impedance units has, de facto, become the SI realization.

For more than five decades, the traceability chain most commonly used for the realization of impedance units has relied on calculable resistance standards. The SI value of DC resistance is established through the quantum

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Hall effect. Subsequently, the resistance and the associated time constant at a specified frequency are inferred from the resistor's geometrical dimensions and the material properties of its constituent elements [125, 126]. The resulting resistance standard is then compared with a capacitance standard at a frequency typically near 1 kHz.

There is an increasing demand for impedance traceability over a broad frequency range, driven by commercial instrumentation spanning from sub-mHz frequencies in electrochemical impedance spectrometers to beyond 100 MHz in high-frequency impedance analyzers.

Helpful review articles on determining the frequency dependence of resistance and capacitance standards are available here [10].

The next sections summarize the various types of impedance standards with a calculable frequency dependence.

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5.1 Resistance standards

Early works in the kHz frequency range can be found here [125–130]. More recently, improvement in the determination of the frequency dependence in the extended audio frequency range can be found in [131–143].

Work on resistance standards having a calculable frequency dependence up to 10 MHz [121, 144–147] and up to 100 MHz and above [148–150]. In addition, very recent work by Schurr, Judaschke and Awan has used miniature thin-film surface-mount-device (SMD) resistors. Instead of calculating relatively large parasitic corrections in a long-wire standard, the physical dimensions are made sufficiently small that parasitic inductance and capacitance and hence the AC/DC correction—are intrinsically small. Standards near 12 906 Ω have been

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demonstrated with calculable ppm-level frequency dependence up to 2 MHz [151, 152]. Schurr and Awan subsequently demonstrated precision measurements of such standards to 2 MHz, including explicit treatment of 4TP, 2TP and cable effects [153].

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5.2 Capacitance standards

Different geometrical designs of capacitance standards have been used over time: Thompson-Lampard type calculable capacitors can be found here [120, 154–158], toroidal cross-capacitors [159–162], coaxial capacitors [121, 163–165].

Different techniques have been developed to determine the frequency dependence of capacitance standards: using Thompson–Lampard reference capacitor [166], using multi-frequency quadrature bridges [121, 167, 168], sampling technique [49, 66, 169], using impedance analysers or VNAs [170–181]

The frequency dependence of capacitance standards in the low frequency range are mainly related to dielectric layers on capacitance plates [182] and at higher frequencies it is generally related to the series inductance and shunt capacitances.

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5.3 Inductance standards

Only a limited number of publications address the computed frequency dependence of inductance standards [183, 184]. In [121] a 4TP coaxial calculable inductance standard was reported with calculated frequency dependence up to 100 MHz being reported. This type of standard has not been constructed/measured, therefore remains a theoretical concept at present [4]. In contrast, using full digital bridges, the frequency dependence of inductance standards can be determined by comparison to a reference resistance standard having a known frequency dependence [48].

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